**Homework 9**

**P20.1.4** Evaluate the following convolution operations: (a)  (b)  (c)  (Note that sinh*x* = −*j*sin*jx*).

**Solution:** (a) sin*ωt*cos*ωt* = 

= .

(b) cos*ωt*cos*ωt* = =cos*ωt* + .

(c) sinh*at* \* sin*ωt* = = = =

= .

**P20.1.9** The switch in Figure P20.1.9 is moved at *t* = 0 from position ‘a’ to position ‘b’ after being in position ‘a’ for a long time. Determine *i* for *t* ≥ 0, assuming  V.



**Solution:** The response to *vSRC* will be determined first, assuming zero initial conditions, and then the initial conditions added by superposition. To determine the response to *vSRC* by convolution, it is necessary to determine the impulse response.



The circuit for determining the response to a unit impulse is shown. In response to the impulse, the inductor behaves as an open circuit, the impulse appears across the inductor, resulting in a current step of 1/3 A; *τ* = 3/6 = 0.5 s. It follows that A. Using the convolution integral, = . Initially, the current in the 1 H inductor is zero, and that in the 2 H inductor is 5 A. At *t* = 0+, the current in the two inductors is equalized in accordance with conservation of flux linkage. At *t* = 0-, the flux linkage is 10 Wb-T; at *t* = 0+, *i*(0+) = 10/3 A, and . The total current is:  A.

**P20.1.10** An input defined as: *vI* = (*t* +1) V, -1 ≤ *t* ≤ 0, *vI* = (-*t* +1) V, 0 ≤ *t* ≤ 1, and *vI* = 0 elsewhere, is applied to a circuit having the impulse response . Determine the output for all *t*.

**Solution:** For   .

For  = .

**P20.1.12** The response of a linear, time-invariant circuit to a unit step input, *u*(*t*), is . Determine the response to an input .

**Solution:** The response *h*(*t*) to a unit impulse the time derivative of the response to a unit step, that is . The required response is: = , for *t* ≥ 0-, or .



**P20.2.1**  Given *h*(*t*) and *x*(*t*) in Figure P20.2.1. Determine *h*(*t*)\**x*(*t*) at *t* = 8.1 s.

**Solution:** When *h*(*t*) is folded around the vertical axis and shifted by 8.1 s to the right, there will be an overlap of 0.4 s, as indicated by the shaded area. This area



is 20×20×0.4 = 160.



**P20.2.2** Given *f*(*t*) in Figure P20.2.2. Determine *f*(*t*)\*2*u*(*t*) at *t* = 5 s.

**Solution:** *f*(*t*)\*2*u*(*t*) = twice the area of *f*(*t*) = area of the rectangle 3×4 = 12.



**P20.2.3** Given *h*(*t*) and *x*(*t*) in Figure P20.2.3. Determine *h*(*t*)\**x*(*t*) at *t* = 6 s.

**Solution:** When *x*(*t*) is folded around the vertical axis and shifted by 6 s to the right, the functions will be as shown. *h*(*λ*) = 3*λ*; *x*(-*λ*) = -(*λ* – 6). The convolution integral becomes:   108.



**P20.2.4** Given the function  and *f*(*t*) in Figure P20.2.4. Determine *h*(*t*)\**x*(*t*) at *t* = 3 s. Verify the result by direct integration.



**Solution:** *Method 1* – *Graphical Solution*: If the pulse is folded around the vertical axis and shifted to the right by 3 s to the right, the figure becomes a shown. The convolution integral, shown shaded, is: .



*Method 2* – *Graphical Solution*: If the exponential function is folded around the vertical axis and shifted



to the right by 3 s, it becomes as shown. The exponential function becomes . The convolution integral, shown shaded, is: , as before.



*Method 3* – *Graphical Solution*: *f*(*λ*) can be expressed as the sum of two step functions: 2*u*(*λ*) – 2*u*(*λ* – 2). Convolving  with the step 2*u*(*λ*) gives: . Convolving  with the step -2*u*(*λ –* 2) involves negating the function and replacing *t* by (*t* – 2), that is, the convolution integral becomes . Adding the two integrals gives: . At *t* = 3 s, the result is the same as before.

*Method 4* – *Analytical Solution*: *f*(*λ*) = 2*u*(*λ*) – 2*u*(*λ* – 2); *x*(*t* – *λ*) = . The first integral is: . The second integral is:  = . Adding the two integrals gives: . At *t* = 3 s, the result is the same as before.